Lecture 15: Resonant Tunneling Diode

- Operation
- Bi-stable switch
- Oscillator
- Fundamental Physics
- Application Example
Resonant Tunneling Diode

An RTD consists of two large bandgap material forming a quantum well with quasi-bound states (in z-direction):

Energy and k conserved during tunneling – negative differential resistance
Resonant Tunneling Diode

Experimental Data (T=300K)

- Important DC figure of merits:
  - Peak current density ($J_p$)
  - Peak Voltage ($V_p$)
  - Valley current density ($J_v$)
  - Valley Voltage ($V_v$)
  - Peak-to-valley ratio: $J_p/J_v$

Plateau is an artifact from oscillations in the bias network.

Valley current is larger than 0 in a real device:

- Thermionic emission
- Field-assisted tunneling
- Transport through 2nd subband
- Elastic/inelastic scattering
Application: Bistable Switch

The two stable bias-points:

Associated with the double barrier is a capacitance:

Assume RTD biased at $V_1 = V_p$ – small increase in $V_s$ forces new biasing point $V_f$

Charge stored over $C$ will increase:

$$\Delta Q = C(V_f - V_p)$$
This charge has to be supplied through $R_L$ – delay in $V_1$.

$$\Delta Q = C(V_f - V_p)$$

Switching time:
$$\tau = \frac{C}{\int_{V_p}^{V_f} \frac{1}{J_{RL}(V_1) - J_{RTD}(V_1)} dV_1}$$

Area inbetween curves gives current to charge $C$ with $\Delta Q$.

$$\tau \geq \frac{C}{\int_{V_p}^{V_f} \frac{1}{J_p - J_v} dV_1} = \frac{C(V_f - V_p)}{J_p - J_v} = \frac{C}{J_p} \frac{V_f - V_p}{1 - J_v / J_p}$$

Fast switching requires a large $J_p$!

**Speed Index**

1/PVR
AlSb/InAs/AlSb Switch Transition

1.7 pS switching of ~ 0.7V

*Slew-rate* of 300mV/pS

$t_b = 1.8$ nm  
$t_w = 7.5$ nm  
10 nm drift region after RTD  
$\tau \sim 1.5$ pS

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**Fig. 2.** Current-voltage characteristic of an InAs/AlSb RTD.

**Fig. 3.** 1.7-ps switching transition time is measured using electro-optic sampling techniques.

E. Özbay, EDL 1993
The RTD has negative differential resistance (NDR):

\[ r = \left. \frac{di}{dv} \right|_{V=V_{NDR}} < 0 \]

If \( R_s \) (series resistance in diode) \( \sim 0 \), the equivalent circuit is a simple RLC:

\[ i(t) \approx i_0 e^{-\frac{g_d}{C} t + j \sqrt{\frac{1}{LC}} t} \]

If \( g_d \) is negative, the oscillations amplitude grows instead of decays.
An RTD biased in the NDR region will oscillate between \( I_p \) and \( I_v \).
High Frequency Oscillators

\[ f_{osc} \approx \frac{1}{2\pi \sqrt{LC}} \]

The oscillation frequency is essentially set by the effective LC-network.
Can oscillate as long as \( \text{Re}(Z_{in}) < 0 \)

\[ f_{\text{max,osc}} = \frac{1}{2\pi C} \sqrt{-\frac{g_d}{R_s} - g_d^2} \]

\[ g_d = \frac{I_p - I_v}{V_p - V_v} \]

For a high \( f_{\text{max}} \), we want small \( R_s \), small \( C \) and large \( g_d \)

Highest RTD \( f_{\text{max}} \sim 850 \text{ GHz} \), which is the highest fundamental mode oscillator made as of today.

\[ P_{out} \propto \frac{1}{\omega^2} \]

Low output power limits applications.
Calculation of current

Assume fully coherent transport:

\[ I^+ = qn^+ \langle v^+ \rangle = 2q \sum_{k_x} v(k) f(E_{F,s}, E(k)) T(k) = \frac{q}{\pi} \int_0^\infty f(E_{F,s}, E(k)) v(k) T(k) dk \]

\[ J^+ = qn^+ \langle v^+ \rangle = 2q \sum_{k_x k_y k_z} v_z(k) f(E_{F,s}, E(k)) T(k) = \frac{2qmkT}{\hbar^3} \int_0^\infty T(E_z) \ln(1 + \exp(E_{f,l} - E_z)) dE_z \]

Where \( T(E_z, V_{sd}) \) is the transmission probability of the quantum system.

\[ E_z = \frac{\hbar^2 k_z^2}{2m} \]
Transmission Probability

Assuming plane waves

\[ \varphi(z) = \begin{cases} 
A e^{ikz} + Be^{-ikz} & I \\
C e^{\gamma z} + De^{-\gamma z} & II \\
E e^{ikz} + Fe^{-ikz} & III \\
... & ... \\
I e^{ikz} & V 
\end{cases} \]

Match wavefunction and its derivative at all interfaces:

\[ \varphi_I(a^-) = \varphi_{II}(a^+) \]

\[ \left. \frac{1}{m^*_I} \frac{\partial \varphi_I(z)}{\partial z} \right|_{z=a^-} = \left. \frac{1}{m^*_II} \frac{\partial \varphi_{II}(z)}{\partial z} \right|_{z=a^+} \]

Solve for \( A \) ... \( I \) (e.g. Using the transfer matrix method)

\[ T(E) = \frac{|I|^2}{|A|^2} \]
At resonance for a symmetric system
\( T = 1 \)
Finite lifetime in well gives rise to a broadening of the state
\( \Gamma_t = \text{FWHM} \)

Around the transmission peak, \( T(E_z) \) is essentially Lorenzian.

\[
T(E_z) \approx \frac{\Gamma_n / 2}{\Gamma_n^2 / 4 + (E_z - E_n)^2}
\]

\[
\Gamma_n \approx \sqrt{\frac{2\hbar^2 E_n T_1^2}{m^* b^2 R_1}}
\]

\( T_1 \) is the transmission probability through a single barrier

\[
J^+ = \frac{2qmkT}{\hbar^3} \int_0^\infty T(E_z) \ln(1 + \exp(E_{f,l} - E_z)) dE_z
\]

If we want \( J^+ \) to be large – \( T(E_z) \) should be 'wide' in energy!

Build RTD with thin barriers and a thin well!
Design example: AlGaAs/GaAs/InGaAs RTD

An indium-rich notch lowers 1st subband, and increases 2nd – higher peak to valley ratio

- **Graph 1:** Strong dependency on barrier thickness
- **Graph 2:** Peak Current Density (kA/cm$^2$) vs. Collector Voltage (V)
- **Graph 3:** Quantum well energy (eV) vs. In (%) in notch
- **Graph 4:** Current Density (kA/cm$^2$) vs. Barrier Thickness (nm)

- **Equation:** $J = 4.69 \times 10^6 \times 10^{(-1.194x)}$
Application Example: Ultra Wideband Source

Startup

\[ v(t) = \frac{2v_o}{\sqrt{1 + \left(\left(\frac{2v_o}{v(0)}\right)^2 - 1\right)e^{-\epsilon\omega_0 t}}} \cos(\omega_0 t + \varphi(0)) \]

\[ \epsilon = -\left( g_{oeq}(0)\sqrt{\frac{L_{eq}}{C_{eq}} + \frac{1}{Q_{tank}}} \right) \]

Decay

\[ v(t) = V_{max} e^{\frac{-\omega_0}{2Q_{tank} PDC}} t \cos(\omega_0 t + \varphi(0)) \]

Very quick turn on/off due to the high slew-rate.
Application Example: Ultra Wideband Source

Gated Resonant Tunnel diode

Inductor
2 Gpulses/s On-Off Keying at 60 GHz

Potential applications:
- UWB communication systems (Gbit/s over short ranges)
- Pulse-based radar systems
- THz spectroscopy (short pulse – large bandwidth)
Large research effort at universities and industry during 1990-2000  
At that time: $f_t f_{max}$ of HEMT/HBTs were ~ 100 GHz, RTD had $f_{max}$ ~ 700 GHz!

RTD – unipolar two-terminal component, with added functionality due to the NDR  
Simple design gives small capacitance $\rightarrow$ fast switching, high oscillation frequency  

Today – $f_t f_{max}$ of transistors are approaching THz, limited usage for RTDs  

Unique $IV$ – can lead to simpler circuit solutions, e.g. oscillators as compared with traditional transistor technology.