Lecture 12: HEMT AC Properties

• Quasi-static operation
• Transcapacitances
• $y$-parameters
• Non-quasi Static effects
• Parasitic resistances / capacitances
• $f_t$, $f_{\text{max}}$

Reading guide for chapter 6: 371-386, 395-407, {407-416 – skip MESFET parts}, 417-443. (i.e. skip any MESFET parts, briefly read NQS parts)
Drift + Continuity – nonlinear, no general analytic solution exists

\[
\mu_n \frac{\partial}{\partial x} \left[ u_{ch} \frac{\partial u_{ch}(x,t)}{\partial x} \right] = \frac{\partial}{\partial t} u_{ch}(x,t)
\]

Assume quasi-static solutions: channel potential (and thus the channel charge) reacts instantiously to any changes in \(v_{ds}, v_{gs}\).

**FIGURE 6.2.** Channel potential profile predicted in a quasi-static analysis for the HFET of Fig. 6-1.
Non-Quasi-Static Solution

This approach is only really accurate for timescales longer than the intrinsic transit time.

Charges has to be supplied from the source – transient involves a charge-front.

QS is still a very useful approximation!

\[ \tau_{tr,QS} = \frac{|Q_{ch}|}{I_D} \]

\[ \mu_n \frac{\partial}{\partial x} \left[ u_{ch} \frac{\partial}{\partial x} u_{ch}(x,t) \right] = \frac{\partial}{\partial t} u_{ch}(x,t) \]

\[ u_{ch}(0,t) = v_{gs}(t) - V_T \]

\[ u_{ch}(0,t) = \alpha \cdot u_{ch}(0,t) \]

FIGURE 6-3. Channel potential profile calculated in a non-quasi-static analysis for the HFET of Fig. 6-1. The charge front is the tip location of the charges moving from the source to the drain.
Terminal charges

Negative channel charges divides onto drain and source charges
Positive charge on gate terminal to make device neutral

Gate charge:

\[ Q_G(t) = WC_{ox} \int_{0}^{L} u_{CH}(x,t) \, dx = u_{CH}(0,t) \int_{0}^{L} \sqrt{1 - \frac{x}{L} \left(1 - \alpha(v_s, v_g, v_d)\right)} = WLC_{ox}'(v_{gs} - V_T) \frac{2(1 + \alpha + \alpha^2)}{3} \]

Definition of drain charge:

\[ i_d(t) = I_D + \frac{\partial Q_D}{\partial t} \]

\[ Q_D(t) = -\frac{WC_{ox}'}{L} \int_{0}^{L} x U_{CH}(x) \, dx = -WLC_{ox}'(v_{gs} - V_T) \frac{6\alpha^3 + 12\alpha^2 + 8\alpha + 4}{15(1 + \alpha)^2} \]

\[ V_T = -0.5V \]

In saturation:

\[ \alpha = 0 \]

\[ Q_D/Q_S = 40/60 \]
Transcapacitances

\[
C_{gg} = \frac{\partial Q_G}{\partial V_G} = WLC_{ox}^\prime \frac{2}{3} \frac{1 + 4\alpha + \alpha^2}{(1 + \alpha)^2}
\]

\[
C_{gd} = -\frac{\partial Q_G}{\partial V_D} = WLC_{ox}^\prime \frac{2}{3} \frac{2\alpha + \alpha^2}{(1 + \alpha)^2}
\]

\[
C_{dg} = -\frac{\partial Q_D}{\partial V_G} = WLC_{ox}^\prime \frac{2}{15} \frac{2 + 14\alpha + 11\alpha^2 + 3\alpha^3}{(1 + \alpha)^3}
\]

\[
C_{dd} = \frac{\partial Q_D}{\partial V_D} = WLC_{ox}^\prime \frac{2}{15} \frac{8\alpha + 9\alpha^2 + 3\alpha^3}{(1 + \alpha)^3}
\]

Note that \( C_{dg} \neq C_{gd} \)
2 minutes excercise

\[ C_{gd} = -\frac{\partial Q_G}{\partial V_D} = WLC_{ox}' \frac{2}{3} \frac{2\alpha + \alpha^2}{(1+\alpha)^2} \]

\[ C_{dg} = -\frac{\partial Q_D}{\partial V_G} = WLC_{ox}' \frac{2}{15} \frac{2+14\alpha + 11\alpha^2 + 3\alpha^3}{(1+\alpha)^3} \]

What is the physical reason that in saturation \( C_{gd} = 0 \) and \( C_{dg} > 0 \)??
Large and small signal Model – Common Source

\[ i_g(t) = \frac{dQ_g(v_g(t), v_d(t), v_s(t))}{dt} = C_{gg}(v_{gs}, v_{ds}) \frac{dv_{gs}}{dt} - C_{gd}(v_{gs}, v_{ds}) \frac{dv_{ds}}{dt} \]

\[ i_d(t) = I_D(v_{gs}, v_{ds}) + \frac{dQ_D(v_g(t), v_d(t), v_s(t))}{dt} = I_D - C_{dg}(v_{gs}, v_{ds}) \frac{dv_{gs}}{dt} + C_{dd}(v_{gs}, v_{ds}) \frac{dv_{ds}}{dt} \]

Common source:

\[ \frac{dv_1}{dt} = 0 \quad \frac{dv_d}{dt} = \frac{dv_{ds}}{dt} \quad \frac{dv_g}{dt} = \frac{dv_{gs}}{dt} \]

For small signal DC:

\[ i_g = 0 \]

\[ i_D = I_D(v_{GS}, v_{DS}) + \frac{\partial I_D}{\partial V_{GS}} |_{V_{GS}} \Delta v_{gs} + \frac{\partial I_D}{\partial V_{DS}} |_{V_{DS}} \Delta v_{ds} \approx I_D(v_{GS}, v_{DS}) + \frac{W C'_{ox} \mu_n}{L} (V_{GS} - V_T) (1 - \alpha) g_m \Delta v_G + g_d \Delta v_D \]

\[ g_m = \frac{W C'_{ox} \mu_n}{L} (V_{GS} - V_T) (1 - \alpha) \]

\[ g_d = \frac{W C'_{ox} \mu_n}{L} (V_{GS} - V_T) \alpha \]
Quasi-static Common Source Small Signal $y$-parameters

\[ i_g(t) = C_{gg} \frac{dv_{gs}}{dt} - C_{gd} \frac{dv_{ds}}{dt} \]

\[ i_d(t) = g_m v_{gs} + g_d v_{ds} - C_{dg} \frac{dv_{gs}}{dt} + C_{dd} \frac{dv_{ds}}{dt} \]

\[
\begin{bmatrix}
\tilde{i}_g \\
\tilde{i}_d
\end{bmatrix} =
\begin{bmatrix}
y_{gg, QS} & y_{gd, QS} \\
y_{dg, QS} & y_{dd, QS}
\end{bmatrix}
\begin{bmatrix}
\tilde{V}_{gs} \\
\tilde{V}_{ds}
\end{bmatrix}
\]

\[ v_{ds} = \tilde{v}_d e^{j\omega t} \quad v_{gs} = \tilde{v}_g e^{j\omega t} \]

\[ i_d = \tilde{i}_d e^{j\omega t} \quad i_g = \tilde{i}_g e^{j\omega t} \]

\[ y_{gg, QS} = j\omega C_{gg} \]

\[ y_{gd, QS} = -j\omega C_{gd} \]

\[ y_{dg, QS} = g_m - j\omega C_{dg} \]

\[ y_{dd, QS} = g_d + j\omega C_{dd} \]

As with the HBT, we can also transform CS $y$-parameters to CG and CD parameters.
All nine capacitances

\[ \Delta V_G = \Delta V_D = \Delta V_S = \Delta V \]
\[ \Delta Q_G = \Delta Q_D = \Delta Q_S = 0 \]

\[ \Delta Q_s + \Delta Q_d + \Delta Q_g = 0 \]
\[ -C_{sg} - C_{dg} + C_{gg} = 0 \]
\[ -C_{gs} - C_{ds} + C_{ss} = 0 \]
\[ -C_{gd} + C_{dd} - C_{sd} = 0 \]

\[ C_{xy} = \delta_{xy} \times \frac{\partial Q_x}{\partial V_y} \]
\[ \delta_{xy} = 1 \text{ if } x=y \]
\[ \delta_{xy} = -1 \text{ otherwise} \]

Can solve for all \( C_{xy} \)
Small Signal Hybrid-π model

\[
\begin{bmatrix}
\tilde{i}_g \\
\tilde{i}_d
\end{bmatrix} =
\begin{bmatrix}
\ y_{gg, QS} & \ y_{gd, QS} \\
\ y_{dg, QS} & \ y_{dd, QS}
\end{bmatrix}
\begin{bmatrix}
\tilde{v}_{gs} \\
\tilde{v}_{ds}
\end{bmatrix}
\]

\[
y_{gg, QS} = j \omega C_{gg}
\]
\[
y_{gd, QS} = -j \omega C_{gd}
\]
\[
y_{dg, QS} = g_m - j \omega C_{dg}
\]
\[
y_{dd, QS} = g_d + j \omega C_{dd}
\]
Quasi Static transit time

\[
\frac{1}{2\pi \times f} \approx \tau_{rs, QS} = \left| \frac{Q_G}{I_D} \right| = \frac{L^2}{\mu_n (V_{GS} - V_T)} \left[ \frac{4}{3} \frac{1 + \alpha + \alpha^2}{(1 + \alpha)(1 - \alpha^2)} \right]
\]

- We expect non-quasi-static effects to be important for high frequencies
- Or, when the transistor is deep in the linear region — small \(g_m\)!
Small signal non-quasi static

\[ \mu_n \frac{\partial}{\partial x} \left[ u_{ch} \frac{\partial}{\partial x} u_{ch}(x,t) \right] = \frac{\partial}{\partial t} u_{ch}(x,t) \]

\[ \frac{d^2 \tilde{i}_{ch}}{dU_{CH}^2} = jD(\omega, I_D)U_{CH} \tilde{i}_{ch} \]

\[ D = \omega \mu_n \left( \frac{WC_{ox}'}{I_D} \right) \]

\[ \tilde{i}_{ch}(U_{CH}) = C_1 \sqrt{U_{CH}} \hat{i}_{1/3} \left( \frac{2}{3} \sqrt{jDU_{CH}^{3/2}} \right) + C_2 \sqrt{U_{CH}} \hat{i}_{-1/3} \left( \frac{2}{3} \sqrt{jDU_{CH}^{3/2}} \right) \]

\( \hat{i} \) is the modified Bessel function

Assume small signal sinusodial perturbation:

\[ u_{ch}(x,t) = U_{CH}(x) + \tilde{u}_{ch}(x)e^{j\omega t} \]

\[ i_{ch}(x,t) = I_{CH}(x) + \tilde{i}_{ch}(x)e^{j\omega t} \]

Express \( \hat{i} \) as a series expansion, and solve for \( u_{ch}(x) \) from boundary conditions (very tedious algebra, see pp. 481-489)
Non QS: Channel Resistance

\[
y_{gg,NQS} = j\omega C_{gg} + j\omega C_{gd} \frac{j(\omega / \omega_0)(\tau_3(\alpha) - \tau_1(\alpha))}{1 + j(\omega / \omega_0)\tau_1} + \ldots
\]

\[
y_{gd,NQS} = j\omega C_{gd} + j\omega C_{gd} \frac{j(\omega / \omega_0)(\tau_3(\alpha) - \tau_1(\alpha))}{1 + j(\omega / \omega_0)\tau_1}
\]

Essentially all corrections are zero/small if \( \omega \ll \omega_0 \)

After plenty of algebra, one obtains the NQS \( y \)-parameters

\[
\omega_0 = \frac{\mu_n (V_{GS} - V_T)}{L^2}
\]

\[
\tau_1 = \frac{4}{15} \frac{1 + 3\alpha + \alpha^2}{(1 + \alpha)^3} = \omega_0 \frac{-C_{sd}}{g_d}
\]

Except for the channel resistance:

\[
r_{ch} = \text{Re} \left( \frac{1}{y_{gs,NQS}} \right) \approx \frac{1}{g_m} \frac{\left(3\alpha^3 + 15\alpha^2 + 10\alpha + 2 \right) (1 - \alpha)}{10(1 + \alpha)(1 + 2\alpha)^2}
\]

\( r_{ch} \) important when \( g_m \) is small, i.e. Not fully linear region / small \( V_{gs} \)

[Diagram of MOSFET with channel resistance highlighted]
Summary – hybrid $\pi$

**DC**

- $v_G$ to $g_d$
- $g_m \Delta v_G$

**Quasi-Static**

- $g_m (1 - j \omega / \omega_0 \tau_1)$

**Non Quasi-Static**

- $\frac{g_m}{1 + j(\omega / \omega_0) \tau_1}$

In saturation: $\alpha = 0$

- $g_m \left(1 - j \frac{\omega}{\omega_0} \frac{4}{15} \right)v_1$
Parasitics: Source, Drain and Gate Resistance

**Source**

- $R_c$
- $R_{lead,1}$
- $R_{lead,2}$
- $R_c$

**Drain**

**Two parts:** contact resistance
channel resistance *prior* to active region

\[
R_c \approx \sqrt{\frac{R_{SH} \rho_{\sigma C}}{W}} \coth \left( L_c \sqrt{\frac{R_{SH}}{\rho_{\sigma C}}} \right)
\]

\[
R_{lead} = R_{SH} L_{gs.gd} / W
\]

**Gate-metal sheet resistance:**

\[
R_{SHG} = \frac{\rho_G}{\Lambda_G}
\]

**Gate resistance**
(similar to base resistance for a HBT):

\[
R_G \approx \frac{1}{3} \frac{W}{L} R_{SHG}
\]

**Figure 6-32:** Schematic diagram depicting the gate structure used in the analysis of distributed gate resistance. The input gate current flows through the metal and diverts into the semiconductor through the gate capacitance.
Extrinsic and intrinsic $g_d$, $g_m$

When measuring the device transconductance, $g_m$
One measures $g_m = \frac{dI_D}{dV_{gs}}$

The intrinsic device transconductance is $g_{mi} = \frac{dI_D}{dV_{gs'}}$

\[
g_m = \frac{g_{mi}}{1 + g_{mi}R_S + g_{di}(R_S + R_D)} < g_{mi}
\]

\[
g_d = \frac{g_{di}}{1 + g_{mi}R_S + g_{di}(R_S + R_D)} < g_{di}
\]

The measured **extrinsic** $g_m$ is smaller than $g_{mi}$ due to $R_s$, and $R_d$!

(For those of you who have done circuit design, $R_s$ is a source degeneracy resistance)
Parasitics capacitances

All capacitances are have related two-terminal parallell-plate like capacitances

No transistor effect, so $C_{xy,p} = C_{yx,p}$

Parasitic capacitances are in parallell to device capacitances:

\[
C_{gg,t} = C_{gg} + C_{gd,p} + C_{gs,p}
\]

\[
C_{gd,t} = C_{gd} + C_{gd,p}
\]

\[
C_{dg,t} = C_{dg} + C_{dg,p}
\]

\[
C_{dd,t} = C_{dd} + C_{dg,p} + C_{ds,p}
\]

Some parasitic can be the dominating capacitance, e.g. $C_{gd}=0$ for $\alpha=0 \rightarrow C_{gd,t}=C_{gd,p}$
\( f_t, f_{\text{max}} \)

1. Determine \([y]\)
2. Convert to \([z]\)
3. Add resistances and parasitic capacitances
4. Calculate \(h_{21} \rightarrow f_t\)
5. Calculate \(U \rightarrow f_{\text{max}}\)

\[
[y]_s = \begin{bmatrix} j\omega C_{gg,t} & -j\omega C_{gd,t} \\ g_m - j\omega C_{dg,t} & g_d - j\omega C_{dd,t} \end{bmatrix}
\]

\[
C_{gg} = \frac{\partial Q_G}{\partial V_G} = WLC_{ox} \frac{2}{3} \frac{1 + 4\alpha + \alpha^2}{(1 + \alpha)^2}
\]

\[
\frac{1}{2\pi \times f_t} = \frac{C_{gg,t}}{g_m} + \frac{C_{gg,t}}{g_m} \left( R_S + R_D \right) g_d + \left( R_S + R_D \right) C_{dg,t}
\]

\[
h_{21} = 1: f_t
\]

\[
f_{\text{max}} = \sqrt{\frac{f_t}{2\pi R_G C_{gd,t}}} \left( 1 + \frac{2\pi f_t}{C_{gd,t}} \Psi \right) \approx \sqrt{\frac{f_t}{8\pi R_G C_{gd,t}}} \left( 1 + \frac{2\pi f_t}{C_{gd,t}} \Psi \right)
\]

\[
\Psi = \left( R_D + R_S \right) \frac{C_{gg,t}^2 g_d^2}{g_m^2} + \left( R_D + R_S \right) \frac{C_{gg,t} C_{dg,t} g_d}{g_m} + \frac{C_{gg,t}^2 g_d}{g_m^2}
\]

\[
U = 1: f_{\text{max}}
\]